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305. Proposed by JOHN J. QUINN, Ph. D., Scottdale, Pa.

1. Suppose two radii R and R_1 revolve uniformly in the ratio $2:3$. Find the equation of the locus of the intersection of R with the chord drawn from the end of the diameter to the extremity of R_1. 2. If the chord be drawn to the extremity of R, what is the locus of the intersection with R_1? 3. Show how an angle can be trisected by means of this curve.

Solution by G. B. M. ZERR, Ph. D., Parsons, W. Va., and A. H. HOLMES, Brunswick, Maine.

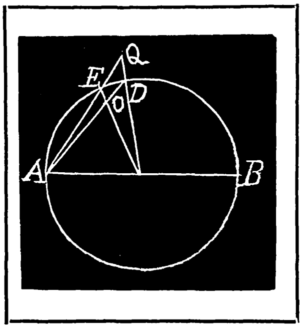
Let $CD=R$, $CE=R_1$. Then $\angle DCB=2\theta$, $\angle ECB=3\theta$, and O is the intersection of BE with R, while Q is the intersection of BD with R_1.

1. Let $CO=\rho$, $CB=a$. Then $CO:CB=\sin CBE:\sin COB$, or $\rho:a=\sin(\frac{1}{2}\pi-\frac{3}{2}\theta):\sin(\frac{1}{2}\pi-\frac{1}{2}\theta)$. $\therefore \rho=a\cos\frac{3}{2}\theta/\cos\frac{1}{2}\theta=a(1-4\sin^2\frac{1}{2}\theta)=a(2\cos\theta-1)$, for locus of O.

2. Let $CQ=\rho'$. $\therefore \rho':a=\sin(\frac{1}{2}\pi-\theta):\sin(\frac{1}{2}\pi-2\theta)$, and $\rho'=a\cos\theta/\cos 2\theta$, for locus of Q.

3. Through any point O, on the curve, draw BO and produce it to meet the circumference in E. Then $\angle ECO=\frac{1}{3}\angle ECB$.

Through any point Q, on the curve, draw QB intersecting the circumference in D. Then $\angle QCD=\frac{1}{3}\angle QCB$.



CALCULUS.

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231. Proposed by PROF. EVA S. MAGLOTT, Ada, Ohio.

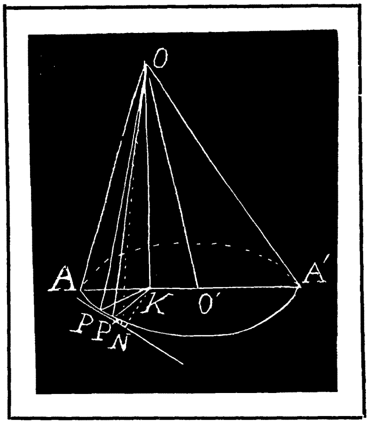
If a right circular cone stand on an ellipse, prove that the convex surface of the cone is $\frac{1}{2}\pi(OA+OA')(OA.OA')^{\frac{1}{2}}\sin\alpha$, where O is the vertex of the cone, A and A' the extremities of the major axis of the ellipse, and α is the semi-angle of the cone at the vertex, using the formula $ds=\frac{1}{2}\rho\sqrt{(\rho^2+p^2)}\,d\theta$, where p is the perpendicular from the vertex to the base of the cone, ρ the distance from the foot of the perpendicular to any point in the perimeter of the base, and θ the angle between the major axis and ρ.

Solution by BENJ. F. FINKEL, Ph. D., Drury College, Springfield, Mo.

The problem as proposed assumes that the secant of the angle between an element of the convex surface and its projection on the base is $\sqrt{(p^2+\rho^2)}/\rho$. This assumption is incorrect, and therefore the convex surface cannot be found by means of the given formula. The correct formula is $ds=\frac{1}{2}\rho^2\,d\theta\sec\gamma$, where γ is the angle between an element of the convex surface and its projection on the base of the cone. The solution is as follows:

Let $O-APA'$ be the cone, axis OO', and OK the perpendicular from the vertex to the base. Let $\alpha=\angle A'OO'=\angle AOO'$; $\beta=\angle O'OK$; $\theta=\angle AKP$; $KO=p$; $KP=\rho$; $OPP'=$an element of the convex surface; and KPP' its projection on the base of the cone.

Area of $KPP'=\frac{1}{2}\rho^2 d\theta$; $\therefore$ area $OPP'=\frac{1}{2}\rho^2 d\theta\sec\gamma$. From the triangles $O'OP$ and $O'KP$, we have $O'O^2+OP^2-2O'O.OP\cos\alpha=O'K^2+KP^2+2O'K.KP\cos\theta$; whence, substituting and reducing, we have



$$\rho\sin\beta\cos\theta=p\cos\beta-\surd(p^2+\rho^2)\cos\alpha\ldots(1).$$

Tangent of angle, ψ, between ρ and the tangent PP' is $\frac{\rho d\theta}{d\rho}$. Let KN be the perpendicular from K on PN. Then angle $PKN=\psi-90°$.

$\therefore KN=\rho\cos(\psi-90°)=\rho\sin\psi$

$$=\rho.\frac{\rho d\theta}{\surd(d\rho^2+\rho^2d\theta^2)}=\frac{\rho^2d\theta}{\surd(d\rho^2+\rho^2d\theta^2)}.$$

$$\sec\gamma=\frac{ON}{KN}=\sqrt{p^2+\left(\frac{\rho^2d\theta}{\surd(d\rho^2+\rho^2d\theta^2)}\right)^2}\div\frac{\rho^2d\theta}{\surd(d\rho^2+\rho^2d\theta^2)}$$

$$=\frac{1}{\rho^2}\sqrt{\rho^2\left(\frac{d\rho}{d\theta}\right)^2+\rho^2(p^2+\rho^2)}.$$

This must be taken with a negative sign, since $\sec\gamma$ diminishes as ρ increases.

$$\therefore dS=\tfrac{1}{2}\rho^2d\theta\sec\gamma=-\tfrac{1}{2}\sqrt{\rho^2\left(\frac{d\rho}{d\theta}\right)^2+\rho^2(p^2+\rho^2)}\,d\theta\ldots(2).$$

From (1), $\dfrac{d\rho}{d\theta}=\dfrac{\rho\surd(p^2+\rho^2)\sin\beta\sin\theta}{\rho\cos\alpha+\surd(p^2+\rho^2)\sin\beta\cos\theta}$.

Substituting in (2) and reducing, we have $dS=$

$$-\tfrac{1}{2}p\sin\alpha\left[\frac{\rho d\rho}{\surd[2p\surd(p^2+\rho^2)\cos\alpha\cos\beta-\rho^2(\cos^2\alpha-\sin^2\beta)-p^2(\cos^2\alpha+\cos^2\beta)]}\right].$$

Let $p^2+\rho^2=x^2$. Substituting and reducing, we have

$$dS=-\tfrac{1}{2}p\sin\alpha\left[\frac{xdx}{\surd[-\cos(\alpha-\beta)\cos(\alpha+\beta)x^2+2px\cos\alpha\cos\beta-p^2]}\right].$$

The limits of ρ are AK and $A'K$. Hence, the limits of x are $\surd(p^2+AK^2)=AO$ and $\surd(p^2+A'K^2)=A'O$. Hence, the convex surface of the cone is

$$S=2\int_{AO}^{A'O}-\tfrac{1}{2}p\sin\alpha\left[\frac{xdx}{\sqrt{[-\cos(\alpha-\beta)\cos(\alpha+\beta)x^2+2px\cos\alpha\cos\beta-p^2]}}\right]$$

$$=-p\sin\alpha\left[\frac{\sqrt{[-\cos(\alpha-\beta)\cos(\alpha+\beta)x^2+2px\cos\alpha\cos\beta-p^2]}}{-\cos(\alpha-\beta)\cos(\alpha+\beta)}\right.$$

$$+\frac{p\cos\alpha\cos\beta}{\cos(\alpha-\beta)\cos(\alpha+\beta)}\left(\frac{1}{\sqrt{[\cos(\alpha-\beta)\cos(\alpha+\beta)]}}\right.$$

$$\left.\left.\times\sin^{-1}\left(\frac{\cos(\alpha-\beta)\cos(\alpha+\beta)x-p\cos\alpha\cos\beta}{p\sin\alpha\sin\beta}\right)\right]_{AO}^{A'O}$$

$$=-\frac{p^2\sin\alpha\cos\alpha\cos\beta}{\cos(\alpha-\beta)\cos(\alpha+\beta)\sqrt{[\cos(\alpha-\beta)\cos(\alpha+\beta)]}}\left[\tfrac{1}{2}\pi-\tfrac{3}{2}\pi\right]$$

$$=\frac{p^2\sin\alpha\cos\alpha\cos\beta\,\pi}{\cos(\alpha-\beta)\cos(\alpha+\beta)\sqrt{[\cos(\alpha-\beta)\cos(\alpha+\beta)]}}=\frac{A'O.AO\,\pi\sin\alpha\cos\alpha\cos\beta}{\sqrt{[\cos(\alpha-\beta)\cos(\alpha+\beta)]}}.$$

Since $A'O\cos^2(\alpha+\beta)=AO\cos(\alpha-\beta)$, $\dfrac{A'O}{AO}=\dfrac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)}$.

$$\frac{A'O+AO}{AO}=\frac{2\cos\alpha\cos\beta}{\cos(\alpha+\beta)}.\quad \text{Also}\ \frac{A'O+AO}{A'O}=\frac{2\cos\alpha\cos\beta}{\cos(\alpha-\beta)}.$$

$$\therefore\ \frac{(A'O+AO)^2}{A'O.AO}=\frac{4\cos^2\alpha\cos^2\beta}{\cos(\alpha-\beta)\cos(\alpha+\beta)},\ \text{or}\ \frac{\cos\alpha\cos\beta}{\sqrt{[\cos(\alpha-\beta)\cos(\alpha+\beta)]}}$$

$$=\tfrac{1}{2}\frac{A'O+AO}{\sqrt{(A'O.AO)}}.$$

$$\therefore S=\tfrac{1}{2}\pi\sqrt{(A'O.AO)}\,(A'O+AO)\sin\alpha.$$

Dr. G. B. M. Zerr sent in two very simple solutions but not by projecting the convex surface on the plane of the elliptic base. Professor Scheffer sent in a solution similar to Dr. Zerr's. As the problem presents no difficulty when referred to rectangular axis with axis of the cone as one of the axes of coordinates, these solutions are omitted. The rectangular equation of the cone referred to planes $A'OA$, APA', and a plane through OK perpendicular to $A'OA$ is $y^2=\sec^2\alpha[(p-z)\sin(\alpha+\beta)+x\cos(\alpha+\beta)][(p-z)\sin(\alpha-\beta)-x\cos(\alpha-\beta)]$.

MECHANICS.

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194. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

A body has a plane face resting on a rough wedge. The wedge is on a rough inclined plane, thick end down and thin edge horizontal. Find the condition that the body will slide down the wedge with constant acceleration, the wedge not slipping the while. Discuss the case in which the angle of friction for wedge and plane is greater than the angle of inclination of the plane.